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EFFECTS OF CHEMICAL REACTION ON FREE CONVECTION FLOW THROUGH A POROUS MEDIUM BOUNDED BY A VERTICAL SURFACE

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The effect of a chemical reaction on a free convection flow through a porous medium bounded by a vertical infinite surface has been studied. Velocity, temperature, and concentration profiles have been obtained for different values of parameters like the Grashof number, Prandtl number, and the chemical reaction parameter in the presence of homogeneous chemical reaction of first order. It is observed that the velocity and concentration increase during a generative reaction and decrease in a destructive reaction. The same is true for the behavior of the fluid temperature. The presence of the porous media diminishes the temperature.

Keywords: *Chemical reaction, porous medium, generative reaction, Eckert number.*

Introduction. The growing need for chemical reactions in chemical and hydrometallurgical industries requires the study of heat and mass transfer with chemical reaction. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of the chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors, as well as metallurgical and chemical engineering. Their other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, and combustion of atomized liquid fuels. The presence of a foreign mass in water or air causes some kind of chemical reaction. This mass may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, for example, polymer production, manufacturing of ceramics or glassware, and food processing. The effect of the presence of a foreign mass on a free convection flow past a semi-infinite vertical plate was studied by Gebhart and Pera [1]. The presence of a foreign mass in air or water causes some kind of chemical reaction. Moreover, during a chemical reaction between two species, heat is generated [2].

A chemical reaction can be codified as either a homogeneous or heterogeneous process. This depends on whether it occurs on an interface or as a single-phase volume reaction. A reaction is said to be first order if its rate is directly proportional to the concentration itself [3]. The effect of chemical reaction on heat and mass transfer in a laminar boundary-layer flow has been studied under different conditions by several authors [4–10]. Diffusion of chemically reactive species in a convective flow along a vertical cylinder has been considered by Ganesan and Rani [11].

The effect of a chemical reaction on a moving isothermal vertical surface with suction has been studied by R. Muthucumaraswamy [12]. Continuing this endevor on the study of a chemical reaction, Kandasamy et al. [13] considered the chemical reaction and thermal stratification effects over a vertically stretching surface. A viscous flow over a nonlinearly stretching sheet in the presence of a chemical reaction and magnetic field has been studied by Raptis et al. [14]. The effect of chemical reaction and heat and mass transfer on a boundary-layer flow over a porous wedge was reported by Kandasamy et al. [15]. Application of a chemical reaction to a micropolar flow over an isothermal vertical cone has been studied by El-Kabeir et al. [16]. Recently, the chemical reaction effect on a mixed convection flow

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along a semi-infinite vertical plate was reported by Mahmoud [17]. The influence of a chemical reaction on a transient MHD free convection flow over a moving vertical plate was investigated by Al-Odat and Al-Azab [18].

During a chemical reaction, heat is generated and in most cases porous media are very widely used to insulate a heated body to maintain its temperature. They are considered to be useful in diminishing the flow temperature. To make the heat insulation of a surface more effective, it is necessary to study a flow through a porous medium and to estimate the effect of chemical reaction on heat and mass transfer. Therefore, here we will attempt to investigate such phenomena. Consequently, the aim of the present work is to study the heat and mass transfer effect on a steady flow of viscous fluid through a porous medium bounded by a porous surface subjected to suction with a constant velocity in the presence of a homogenåous chemical reaction of first order. Examination of such a flow model reveals the influence of chemical reaction on the velocity, temperature, and concentration profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the chemical reaction parameter. It is hoped that these results will not only provide a useful information for application, but also will serve as a complement to the previous studies.

Mathematical Analysis. We consider a steady flow of an incompressible viscous fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface. The $\tilde{o}^*$ axis is taken along the surface in an upward direction and the $\acute{o}^*$ axis is normal to it. The fluid properties are assumed to be constant except for the density in the body force term. A chemically reactive species is emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it undergoes a homogeneous chemical reaction. The reaction is assumed to take place entirely in the stream. The basic equations for the flow through a highly porous medium are

$$\frac{\partial v^*}{\partial y^*} = 0 , \tag{1}$$

$$v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_1 (T^* - T_\infty) + g\beta_2 (C^* - C_\infty) - \frac{\nu u^*}{k_\mathrm{p}} , \tag{2}$$

$$v^* \frac{\partial T^*}{\partial y^*} = \frac{\alpha}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\nu}{c_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 , \tag{3}$$

$$v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - k_\mathrm{c} C^* . \tag{4}$$

Here we have assumed that the level of species concentration is very low; therefore, the heat generated due to the chemical reaction can be neglected.

Equation (1) gives

$$v^* = \mathrm{const} = - v_0 , \tag{5}$$

where $v_0 > 0$ and v^* is the steady normal velocity of suction on the surface. The relevant boundary conditions are

$$u^* = 0 , \quad T^* = T_\mathrm{w} , \quad C^* = C_\mathrm{w} \quad \text{at} \quad y^* = 0 ;$$

$$u^* \to 0 , \quad T^* \to T_\infty , \quad C^* \to C_\infty \quad \text{as} \quad y^* \to \infty . \tag{6}$$

Let us introduce the following nondimensional parameters:

$$u=\frac{u^*}{v_0}\,,\quad y=\frac{v_0 y^*}{\nu}\,,\quad \theta=\frac{T^*-T_\infty}{T_w-T_\infty}\,,\quad C=\frac{C^*-C_\infty}{C_w-C_\infty}\,,\quad \mathrm{Pr}=\frac{\rho\nu c_p}{\alpha}\,,\quad \mathrm{Sc}=\frac{\nu}{D}\,,$$

$$\mathrm{Gr}=\frac{\nu g\beta_1\,(T_w-T_\infty)}{v_0^3}\,,\quad \mathrm{Gc}=\frac{\nu g\beta_2\,(C_w-C_\infty)}{v_0^3}\,,\quad \mathrm{E}=\frac{v_0^2}{c_p\,(T_w-T_\infty)}\,,\quad k=\frac{v_0^2}{\nu^2}\,k_p\,,$$

$$k_0=\frac{\nu}{v_0^2}\,k_c\,. \tag{7}$$

The governing system of equations (1)–(4) with the use of (7) is reduced to the following nondimensional form:

$$u''+u'=-\,\mathrm{Gr}\,\theta-\mathrm{Gc}\,C+k^{-1}u\,, \tag{8}$$

$$\theta''+\mathrm{Pr}\,\theta'=-\,\mathrm{Pr}\,\mathrm{E}\,u'^2\,, \tag{9}$$

$$C''+\mathrm{Sc}\,C'=k_0\,\mathrm{Sc}\,C\,. \tag{10}$$

The corresponding boundary conditions are

$$u=0\,,\quad \theta=1\,,\quad C=1\ \text{ at }\ y=0\,;$$

$$u\to 0\,,\quad \theta\to 0\,,\quad C\to 0\ \text{ as }\ y\to\infty\,. \tag{11}$$

In order to obtain solutions of the coupled nonlinear system of equations (8)–(10) with the boundary conditions (11), we expand u, θ, and C in powers of the Eckert number E (assuming that E is very small), i.e., we write

$$u=u_0+\mathrm{E}\,u_1+\mathrm{O}\,(\mathrm{E}^2)\,,\quad \theta=\theta_0+\mathrm{E}\,\theta_1+\mathrm{O}\,(\mathrm{E}^2)\,,\quad C=C_0+\mathrm{E}\,C_1+\mathrm{O}\,(\mathrm{E}^2)\,. \tag{12}$$

Substituting Eqs. (12) into Eqs. (8)–(10), equating the coefficients at the terms with the same powers of E, and neglecting terms of the order of E^2 and higher orders, we get the following equations:
zero order

$$u_0''+u_0'=-\,\mathrm{Gr}\,\theta_0-\mathrm{Gc}\,C_0+k^{-1}u_0\,, \tag{13}$$

$$\theta_0''+\mathrm{Pr}\,\theta_0'=0\,, \tag{14}$$

$$C_0''+\mathrm{Sc}\,C_0'=k_0\,\mathrm{Sc}\,C_0\,; \tag{15}$$

first order

$$u_1''+u_1'=-\,\mathrm{Gr}\,\theta_1-\mathrm{Gc}\,C_1+k^{-1}u_1\,, \tag{16}$$

$$\theta_1''+\mathrm{Pr}\,\theta_1'=-\,\mathrm{Pr}\,u_0'^2\,, \tag{17}$$

$$C_1'' + \mathrm{Sc}\, C_1' = k_0\, \mathrm{Sc}\, C_1\,. \tag{18}$$

The corresponding boundary conditions are

$$u_0 = 0\,,\;\; u_1 = 0\,,\;\; \theta_0 = 1\,,\;\; \theta_1 = 0\,,\;\; C_0 = 1\,,\;\; C_1 = 0 \;\; \text{at} \;\; y = 0\,;$$

$$u_0 \to 0\,,\;\; u_1 \to 0\,,\;\; \theta_0 \to 0\,,\;\; \theta_1 \to 0\,,\;\; C_0 \to 0\,,\;\; C_1 \to 0 \;\; \text{at} \;\; y \to \infty\,. \tag{19}$$

Solving Eqs. (13)–(18) under the boundary conditions (19) and substituting the obtained solutions into Eq. (12), we can present the solutions for u, θ, and C as follows:

$$\begin{aligned} u\,(y) = {} & (k_5 + k_6)\exp\,(k_3 y) - k_5 \exp\,(-\Pr y) - k_6 \exp\,(-0.5\alpha_1 y) + \mathrm{E}\Bigg[\frac{1}{\alpha_2}\big(-\mathrm{Gr}\,(k_8 + k_9 \\ & + k_{11})\exp\,(k_3 y)\big) + \frac{\mathrm{Gr}\,k_8 \exp\,(k_3 y)}{\alpha_3} + \frac{\mathrm{Gr}\,k_9 \exp\,(k_3 y)}{\alpha_1^2 - \alpha_1 - k_1} + \frac{\mathrm{Gr}\,k_{11} \exp\,(k_3 y)}{\alpha_4} \\ & + \frac{\mathrm{Gr}\,(k_8 + k_9 + k_{11})\exp\,(-\Pr y)}{\alpha_2} - \frac{\mathrm{Gr}\,k_8 \exp\,(-2\Pr y)}{\alpha_3} - \frac{\mathrm{Gr}\,k_9 \exp\,(-\alpha_1 y)}{\alpha_1^2 - \alpha_1 - k_1} \\ & - \frac{\mathrm{Gr}\,k_{11} \exp\,(-(\Pr + 0.5\alpha_1)\,y)}{\alpha_4}\Bigg]\,, \end{aligned} \tag{20}$$

$$\begin{aligned} \theta\,(y) = {} & \exp\,(-\Pr y) + \mathrm{E}\,[-(k_8 + k_9 + k_{11})\exp\,(-\Pr y) + k_8 \exp\,(-2\Pr y) \\ & + k_9 \exp\,(-\alpha_1 y) + k_{11} \exp\,(-(\Pr + 0.5\alpha_1 y)]\,, \end{aligned} \tag{21}$$

$$C\,(y) = \exp\,(-0.5\alpha_1 y)\,. \tag{22}$$

The expressions for the constants involved in Eqs. (20)–(22) are given in the Appendix. Equation (22) for the fluid concentration can be considered under the following conditions: $k_0 > 0$ for a destructive reaction; $k_0 = 0$ in the absence of reaction; $k_0 < 0$ for a generative reaction.

The rate of heat transfer in terms of the Nusselt number is given by

$$\mathrm{N} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \Pr + \mathrm{E}\,[-\Pr\,(k_8 + k_9 + k_{11}) + 2\Pr k_8 + \alpha_1 k_9 + k_{11}\,(\Pr + 0.5\alpha_1)]\,. \tag{23}$$

The nondimensional skin friction at the surface is given by

$$\begin{aligned} \tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = {} & k_3\,(k_5 + k_6) + \Pr k_5 + 0.5\alpha_1 k_6 + \mathrm{E}\Bigg[\frac{1}{\alpha_2}\big(-\mathrm{Gr}\,(k_8 + k_9 + k_{11})\,k_3\big) + \frac{\mathrm{Gr}\,k_8 k_3}{\alpha_3} \\ & + \frac{\mathrm{Gr}\,k_9 k_3}{\alpha_1^2 - \alpha_1 - k_1} + \frac{\mathrm{Gr}\,k_{11} k_3}{\alpha_4} - \frac{\mathrm{Gr}\,\Pr\,(k_8 + k_9 + k_{11})}{\alpha_2} + \frac{2\,\mathrm{Gr}\,\Pr\,k_8}{\alpha_3} \\ & + \frac{\mathrm{Gr}\,k_9 \alpha_1}{\alpha_1^2 - \alpha_1 - k_1} - \frac{\mathrm{Gr}\,k_{11}\,(\Pr + 0.5\alpha_1)}{\alpha_4}\Bigg]. \end{aligned} \tag{24}$$

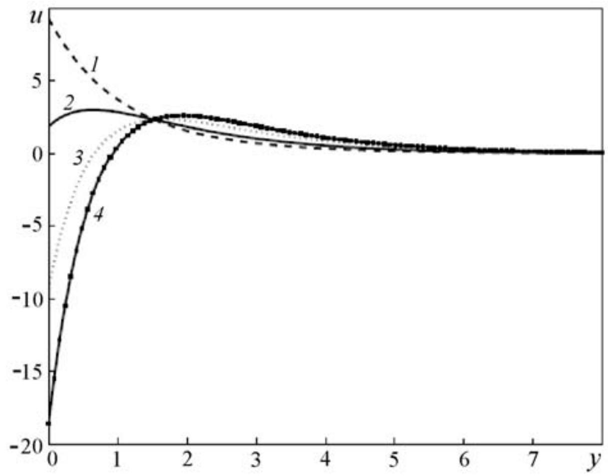

Fig. 1. Velocity profiles at Pr = 0.71, k_0 = 1, k = 1, and Gr = Gc = 5 for different values of E: 1) E = –0.05; 2) –0.01; 3) 0.05; 4) 0.1.

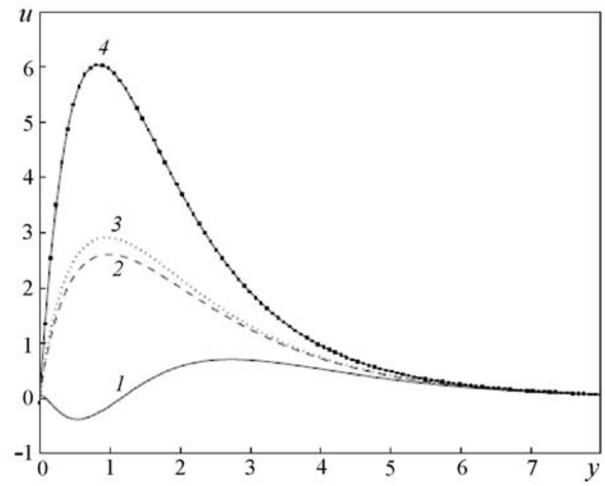

Fig. 2. Velocity profiles at Pr = 7, k_0 = 1, k = 1, and Gr = Gc = 10 for different values of E: 1) E = –0.05; 2) –0.01; 3) 0; 4) 0.1.

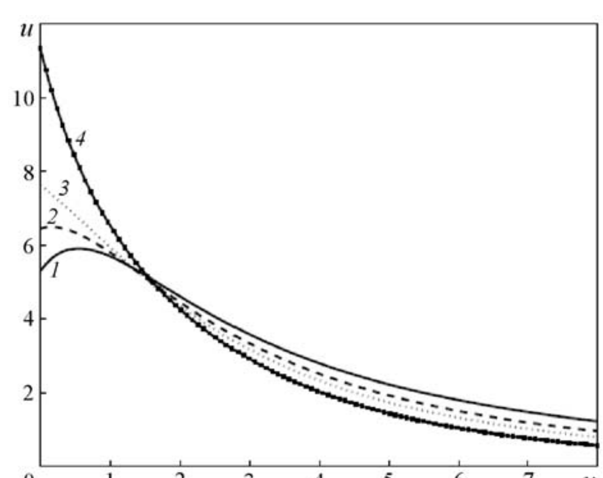

Fig. 3. Velocity profiles at Pr = 0.71, k = 1, E = 0.05, and Gr = Gc = 5 for different values of k_0: 1) k_0 = –0.04; 2) –0.02; 3) 0; 4) 0.05.

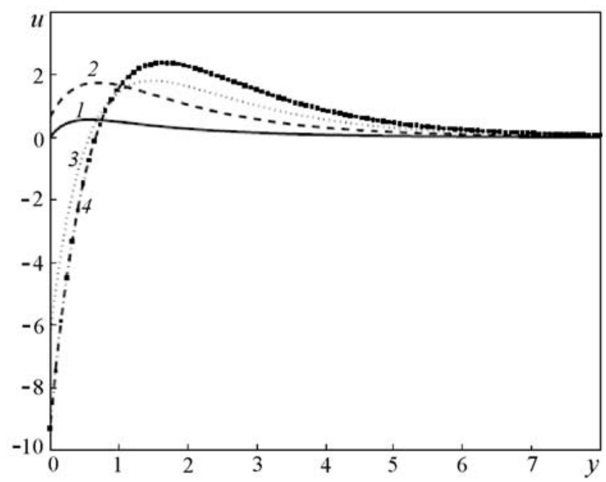

Fig. 4. Velocity profiles at Pr = 0.71, k_0 = 1, E = 0.05, and Gr = Gc = 5 for different values of k: 1) k = 0.1; 2) 0.4; 3) 0.7; 4) 1.

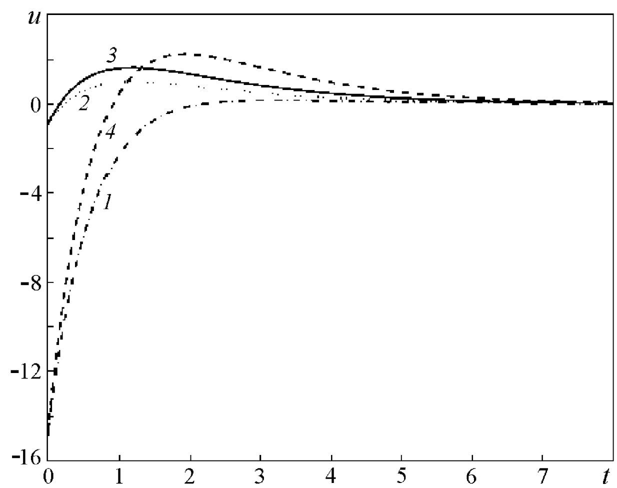

Fig. 5. Velocity profiles at Pr = 0.71, k_0 = 1, k = 1, E = 0.05, and Gc = 5 for different values of Gr: 1) Gr = –2; 2) –0.5; 3) 0.5; 4) 2.

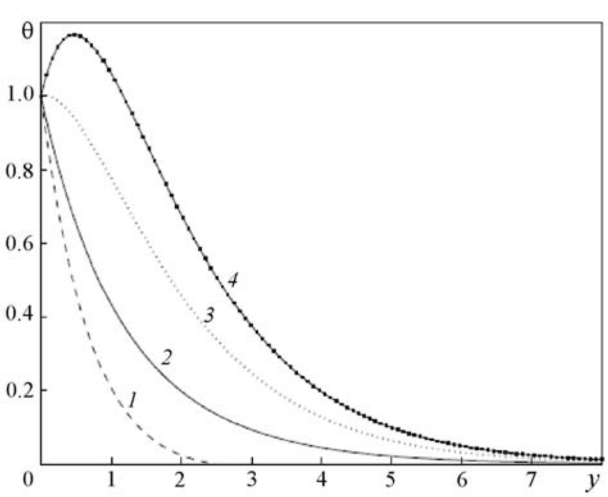

Fig. 6. Temperature profiles at Pr = 0.71, k_0 = 1, k = 1, and Gr = Gc = 5 for different values of E: 1) E = –0.05; 2) –0.01; 3) 0.05; 4) 0.1.

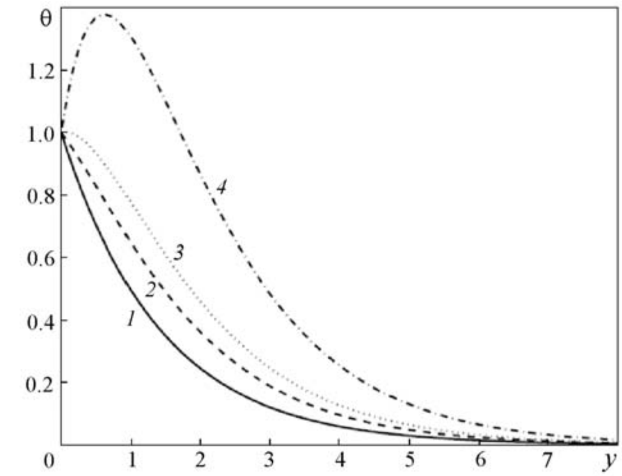

Fig. 7. Temperature profiles at Pr = 0.71, k_0 = 1, E = 0.05, and Gr = Gc = 5 for different values of k: 1) k = 0.1; 2) 0.7; 3) 1; 4) 2.

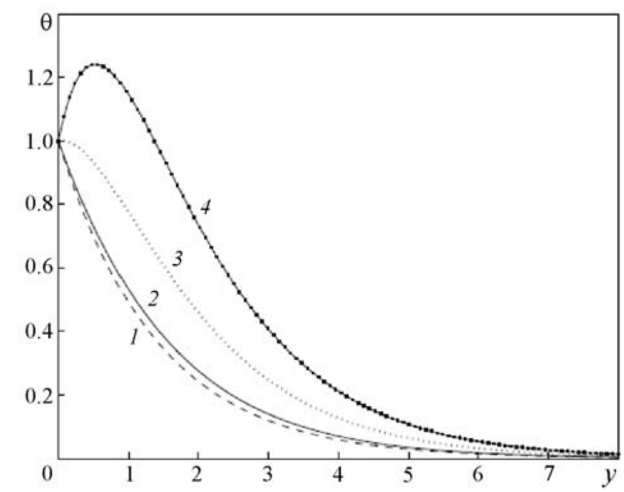

Fig. 8. Temperature profiles at Pr = 0.71, k_0 = 1, k = 1, E = 0.05, and Gc = 5 for different values of Gr: 1) Gr = –5; 2) –1; 3) 5; 4) 10.

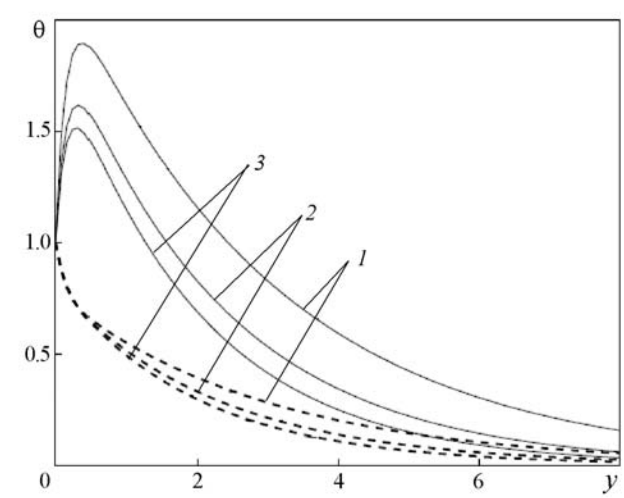

Fig. 9. Temperature profiles at Pr = 7, Gr = Gc = 10, and k = 0 (solid lines) and k = 0.1 (dashed lines) for different values of k_0: 1) k_0 = –0.04; 2) 0; 3) 0.04.

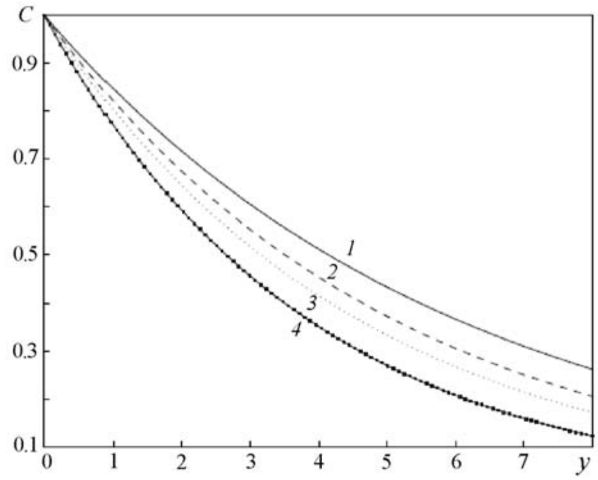

Fig. 10. Concentration profiles at Pr = 0.71, k = 1, E = 0.05, and Gr = Gc = 5 for different values of k_0: 1) k_0 = –0.04; 2) –0.02; 3) 0; 4) 0.05.

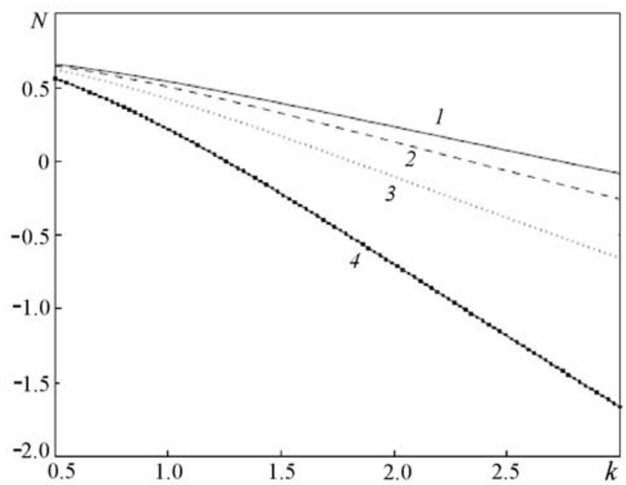

Fig. 11. Nusselt number against the permeability parameter at Pr = 0.71, k_0 = 1, E = 0.01, and Gc = 10 for different values of Gr: 1) Gr = 1; 2) 2; 3) 4; 4) 8.

Results and Discussion. In order to get a clear insight into the physical problem, numerical results are presented in figures. The fluids considered in this study are air and water (Pr = 0.7 and 7.0, respectively, at 20^{o}C). The velocity fields, the temperature and concentration profiles, the Nusselt number, and skin friction are obtained at Sc = 0.22 for different values of other parameters.

Velocity profiles. The velocity profiles are depicted in Figs. 1–5. The effect of the Eckert number on the velocity field is shown in Figs. 1 and 2. It is interesting to observe that the fluid velocity increases and reaches its maximum over a very short distance from the plate and then decreases gradually to zero for positive values of the Eckert number* ($T_w > T_\infty$). This corresponds to a hot fluid layer near the surface and an externally cooled surface. A higher fluid velocity is observed for small Eckert numbers. A negative Eckert number ($T_w > T_\infty$) means that the temperature of the fluid layer near the vertical surface is less than the ambient temperature and therefore the velocity near the vertical surface is higher but then gradually decreases to zero.

Figure 3 shows the effect of the chemical reaction parameter on the velocity profiles. A generative reaction ($k_0 < 0$) increases the fluid flow velocity, whereas a destructive reaction ($k_0 > 0$) reduces it. An increase in the permeability parameter results in an increase in the velocity (see Fig. 4).

*From the Editors. Nondimensional criteria Gr, E, and N are usually positive quantities. Thus, the Editors consider that it could be logical to somehow differently label these parameters introduced by the authors, for example, as E′, so that we could have E = |E′|.

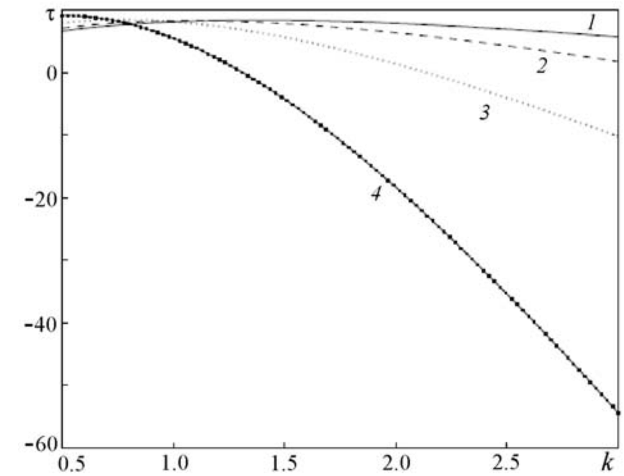

Fig. 12. Skin friction against the permeability parameter at Pr = 0.71, k_0 = 1, E = 0.01, and Gc = 10 for different values of Gr: 1) Gr = 1; 2) 2; 3) 4; 4) 8.

The Grashof number Gr describes the effect of free convection currents, and the case $\mathrm{Gr} > 0$ $(T_w > T_\infty)$ corresponds to an externally cooled surface. It is seen from Fig. 5 that, as Gr increases from 0.5 to 2, the surface becomes cooler and the velocity increases. Similarly, as Gr decreases from –0.5 to –2, the surface becomes hotter and the velocity decreases.

Temperature profiles. A graphical presentation of the thermal boundary layer for different values of the parameters is given in Figs. 6–9. The nondimensional temperature for various values of the Eckert number is plotted in Fig. 6. Clearly the temperature is equal to unity at the wall and to zero in the free stream. For $\mathrm{E} > 0$ $(T_w > T_\infty)$, the temperature increases over the boundary layer and near the surface. For $\mathrm{E} < 0$ $(T_w < T_\infty)$, there is a rapid temperature drop and heat flows into the surface. Here, an inflection point occurs in the curve for E = 0.05, whereas the curve for E = 0.01 displays a hill. An increase in the permeability parameter increases the temperature (Fig. 7). Physically this means that such an increase results in an increase in the velocity. Hence an increase in the wall temperature is observed. In the case of an externally cooled plate $(\mathrm{Gr} > 0)$ the temperature increases near the surface, and for an externally heated plate $(\mathrm{Gr} < 0)$ there is a rapid temperature drop as shown in Fig. 8.

An important aspect of this work is seen in Fig. 9. This figure shows that in the absence of porous media the fluid temperature increases with decrease in the chemical reaction parameter. The presence of porous media diminishes the flow temperature, and heat insulation of the surface becomes more effective. Figure 9 shows also the effect of the chemical reaction parameter on the temperature profiles. When the diffusion species is destroyed in the flow, i.e., a destructive reaction takes place $(k_0 > 0)$, the products have a lower potential energy than the reactants. Therefore, the fluid temperature for k_0 = 0.04 is less than that without chemical reaction (k_0 = 0). Similarly, when the diffusion species is generated in a homogenåous reaction, a generative reaction occurs $(k_0 < 0)$. In this case the product is unstable and its energy state is higher than for the reactants. This corresponds to a higher temperature at k_0 = –0.04 than without a chemical reaction. The situation mentioned is similar for both values of k.

Concentration distribution. Figure 10 shows that a destructive reaction reduces the concentration. This is due to the fact that for $k_0 > 0$ the last term in the mass diffusive equations (15) and (18) becomes positive and it contributes to the concentration reduction. At the same time, the last term in the equations mentioned becomes negative for a generative reaction $(k_0 < 0)$ and, as a result, it leads to a concentration increase. A variation in the heat transfer rate expressed in terms of the Nusselt number N is shown in Fig. 11. It is observed that N decreases with increasing permeability. This decrease becomes stronger for large Grashof number, i.e., for externally very cooled surface. Similarly, the skin friction depicted in Fig. 12 decreases with increase in the permeability parameter and for large Gr the decrease becomes stronger.

CONCLUSIONS

1. The velocity of a fluid flow increases with the Eckert number, permeability parameter, and the Grashof number and decreases with increase in the chemical reaction parameter.

2. In most cases the velocity attains a maximum near the surface and thereafter decreases.

3. An inflection occurs in the temperature profiles for positive values of E, k, and Gr, and the curves for large positive values of the above parameters display hills.

4. There is a fall in the concentration due to the increase in the value of the chemical reaction parameter. The concentration increases in the presence of a generative reaction.

5. A destructive chemical reaction reduces the fluid temperature, and a generative reaction increases it.

6. The excess heat produced during the generative first-order homogeneous chemical reaction is diminished in the presence of a porous medium.

The present study of the physics of a fluid flow over a vertical surface can be utilized as the basis for many scientific and engineering applications and in solving more complex problems for vertical surfaces.

NOTATION

C, nondimensional fluid concentration; C^*, concentration, mol/m^3; C_∞, fluid concentration far away from the wall, mol/m^3; c_p, specific heat at a constant pressure, J/(kg·deg); D, mass diffusivity, m^2/sec; E, Eckert number; Gc, mass Grashof number; Gr, thermal Grashof number; g, gravitational acceleration, m/sec^2; k, nondimensional permeability coefficient of a porous medium; k_0, nondimensional rate of a chemical reaction; k_c, rate of chemical reaction, sec^{-1}; k_p, permeability of a porous medium, m^2; N, Nusselt number; Pr, Prandtl number; Sc, Schmidt number; T_∞, fluid temperature far away from the wall, ^{o}C; T^*, temperature, ^{o}C; u^*, v^* velocity components, m/sec; u, nondimensional velocity; v_0, suction velocity, m/sec; x^*, y^* space coordinates, m; y, nondimensional space coordinate; α, thermal conductivity, W/(m·deg); β_1, coefficient of volume expansion, 1/deg; β_2, coefficient of volume expansion with concentration, m^3/mol; θ, nondimensional temperature; ν, kinematic viscosity, m^2/sec; ρ, fluid density, kg/m^3; τ, nondimensional skin friction. Subscripts and superscripts: w, wall; ∞, far away from the wall; 0 and 1, zero and first orders; prime denotes differentiation with respect to y.

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Appendix

$$k_1 = \frac{1}{k}\,, \quad \alpha_1 = \mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + 4\mathrm{Sc}\,k_0}\,, \quad \alpha_2 = \mathrm{Pr}^2 - \mathrm{Pr} - k_1\,, \quad \alpha_3 = 4\mathrm{Pr}^2 - 2\mathrm{Pr} - k_1\,,$$

$$\alpha_4 = (\mathrm{Pr} + 0.5\alpha_1)^2 - (\mathrm{Pr} + 0.5\alpha_1) - k_1\,, \quad k_3 = -\,0.5\,(1 + \sqrt{1 + 4k_1}\,)\,, \quad k_5 = \frac{\mathrm{Gr}}{\alpha_2}\,,$$

$$k_6 = \frac{\mathrm{Gc}}{0.25\alpha_1^2 - 0.5\alpha_1 - k_1}\,, \quad k_8 = -\,0.5\mathrm{Pr}\,k_5^2\,, \quad k_6' = 0.5k_6\alpha_1\,, \quad k_9 = -\frac{k_6'^2\,\mathrm{Pr}}{\alpha_1^2 - \mathrm{Pr}\,\alpha_1}\,,$$

$$k_{11} = -\frac{2k_5k_6'\,\mathrm{Pr}^2}{0.5\alpha_1\,(\mathrm{Pr} + 0.5\alpha_1)}\,.$$